

## ECONOPHYSICS : A NEW DISCIPLINE

SÓNIA R. BENTES\*

---

*This paper debates the contribution of Econophysics to the economic or financial domains. Since the traditional approach performed by Economics or Finance has revealed to be insufficient in fully characterizing and explaining the correspondingly phenomena, we discuss whether Econophysics can provide a new insight onto these matters. Thus, an assessment is presented in order to weight its potential opportunities and limitations. This is particularly relevant as it is widely recognized that during its yet short existence Econophysics has experienced a growing interest not only by physicists but also by economists in searching for new approaches that could help explaining existing questions. In fact, many papers have been submitted, some books have been released, new journals have been published, several conferences have been held, a site is maintained – <http://www.unifr.ch/econophysics> where news, events, book reviews, papers and a blog are exhibited; a 3-year licentiate studies (University of Silesia [1]) and a B.Sc. course (University of Wroclaw [2]) have been created and also some Ph.D. thesis have been written. Therefore, a fundamental question arises: Is this just a fad or is it something much more consistent that will prevail? This is what this paper addresses.*

**PACS numbers:** 89.65.Gh; 89.90.+n

**Keywords:** Economics; Econophysics; Financial Markets; Interdisciplinary Physics

---

### Origins

In the last decade physicists have been increasingly concerned about economic or financial subjects. This has given way to a new branch of knowledge called “Econophysics”. This neologism was first introduced by H.E. Stanley, in 1996<sup>3</sup>, in an attempt to legitimize the study of Economics by physicists and is a result of the combination of the word “Economics” with the word “Physics”, in a clear analogy with terms such as Biophysics, Geophysics and Astrophysics. Nevertheless, the involvement of Physics in Social Sciences is longer, dating back at least to 1526 when Copernicus, while studying the behaviour of the inflation, established the theoretical foundations of what was later known as the Greshnam law, according to what “bad money drives out good under legal tender laws”. Newton was the following physicist involved

in Economics, who as a warden of the Royal Mint of Great Britain during 1696-1701, standardized Britain’s coinage. Another interesting insight was brought up by Edmond Halley, the famous mathematician and astronomer, who has derived the foundations of life insurance. Based on data relating to births and deaths in the German city of Breslau, between 1687 and 1691, Halley constructed in 1693 his own life table (for individual ages, not for groups), which was found to give a reasonably accurate picture of survival and become well known throughout Europe. Later, in 1738, Bernoulli introduced the idea of utility to describe people’s preferences. Subsequently, Laplace stressed out, in 1812, that events that might seem random and unpredictable can in fact be predictable.

Despite these incursions, the first known attempt to describe this new branch of knowledge was due to Quetelet who, in 1835, coined it “Social Physics”. The notion encountered support in Auguste Comte (1798-1857), who has for the first time considered it as a separate scientific

---

\* ISCAL, Av. Miguel Bombarda 20, 1069-035 Lisboa, Portugal,  
E-mail : [smbentes@iscal.ipl.pt](mailto:smbentes@iscal.ipl.pt)

discipline. This idea would be raised up again by Majorana, in the 20th Century, in his seminal paper on the Analogy between statistical laws in Physics and in Social Sciences, where he outlined the opportunities and drawbacks of applying methods of the former to the latter<sup>4</sup>. Recently, some physicists prefer to call it “Phynance” in a contraction of the terms “Physics” and “Finance”<sup>5,6</sup>, others have adopted the terminology “Financial Physics”; however, the name Econophysics has prevailed since it encompasses a much wider focus ranging from Economics to Finance.

Although Econophysics has emerged from the urge of describing economics or financial phenomena by means of applying methods from the science of Physics, it is worthy to note that the first power-law ever discovered in nature, a most commonly distribution evidenced in Physics,<sup>1</sup> was originally observed in Economics by Pareto<sup>7</sup>, when analyzing the distribution income among the population. Similarly<sup>8</sup>, proposed the first theory of market fluctuation, five years before Einstein’s famous paper on Brownian motion<sup>9</sup>, where he derived the partial differential heat/diffusion equation governing Brownian motion and made the estimate for the size of molecules. Specifically<sup>8</sup>, gave the distribution function for the Wiener stochastic process – the stochastic process underlying Brownian motion – linking it mathematically with the diffusion equation. It is thus telling that the first theory of the Brownian motion was developed to model financial asset prices! These two examples illustrate that the relation between both sciences is bidirectional and not a one-way route, as one might believe, a fact that must be considered when studying this subject. Nevertheless, the impact of Physics in Economics/Finance is much more evident than the reverse.

### **Scope**

Before proceeding any further, additional enlightenment is required in order to fully understand the meaning and implications of the word “Econophysics”. There are, indeed, a number of attempts in literature to define it. Ref. [10, p. 3] considers that “Econophysics is a hybrid discipline (. . . ) that applies various models and concepts originated in Physics to economic (and financial) phenomena.” and adds, in a more eloquent way, that “Econophysics presents itself as a new way of thinking about the economic and financial systems through the ‘glasses’ of physics”. According to Ref. [11, p. 1] this discipline is “a quantitative approach using ideas, models,

conceptual and computational methods of Statistical Physics”. From Mantegna and Stanley point of view [12, p. 355] “the word Econophysics describes the present attempts of a number of physicists to model financial and economic systems using paradigms and tools borrowed from Theoretical and Statistical Physics”.

Basically, the general idea is to use concepts and tools of Physics in order to study economic/financial problems<sup>10-12</sup>. Building on this, physicists have been mainly applying concepts and methodologies of Statistical Physics (*e.g.*, scaling, universality, disordered frustrated systems and self-organized systems) to describe such complex systems as economic or financial systems, as they appear to be. Indeed, most approaches based on the fundamentals of Physics perceive financial/economic phenomena as complex evolving systems<sup>13</sup>. This is due to the multiple interacting components exhibited by the inherent time series, *e.g.*, stock market indexes or inflation rates. In particular, these systems are expressed in the light of their statistical properties and their principles (microscopic models, scaling laws) are used to develop models to explain the corresponding behaviour<sup>10</sup>. Some examples of the application of Statistical Physics to Finance can be found in<sup>14-18</sup>.

### **The Call for a New Discipline**

One fundamental question that may arise when approaching this subject is what has triggered the urge for this new discipline. It seems reasonable that if Econophysics has emerged some underlying factors must have been driving it. When addressing this matter two main reasons seem to apply: (i) the limitations of the traditional approach of Economics/Finance and, (ii) the advantages of the empirical method used in Physics. As for the first, there was already a growing debate in literature about the drawbacks of methods advocated by Economics/Finance. Recently, some of these shortcomings were listed by Ref. [19] in the economic domain. According to the author concepts like utility maximization, perfect competition and diminishing marginal productivity are empirically and logically flawed and should not be used in Econophysics. For instance, a foundation belief of neoclassical Economics is that all individuals want to maximize their subjective utility. However, a considerable number of violations exist that contradicts the neoclassical theory of consumer. One relates to income, another has to do with habit, and a third one refers to the way tastes are formed, transmitted and modified.

Another assumption frequently criticized is the postulate of perfect competition. Indeed, numerous researchers have found that marginal cost, denoted by the marginal benefit of

---

1. Power-laws have received considerable attention in physics because they indicate scale free behaviour and are characteristic of critical or nonequilibrium phenomena.

consumption, is irrelevant to the firm<sup>8</sup>. Finally, the proposition of diminishing marginal productivity is also falsified.

Alternatively, critics to the Finance theory refer basically to the Efficient Market Hypothesis (EMH) formulated<sup>21</sup>, which comprises three major versions: "weak", "semi-strong", and "strong" form. Weak EMH claims that prices on traded assets (*e.g.*, stocks, bonds, or property) already reflect all past publicly available information. Semi-strong EMH both asserts that prices reflect all publicly available information and that prices instantly change to reflect new public information. Strong EMH additionally postulates that prices instantly reflect even hidden or "insider" information. Despite its popularity, this principle is strongly controversial and has been successively questioned, since it represents a mere idealization that can hardly be verified<sup>22-24</sup>. In fact, the idea that markets are rational, from which this theory departs, is a theoretical construction that can be easily violated. Another example stands from the Capital Asset Pricing Model (CAPM), which can not be applied (i) if investors differ in their expectations and, (ii) if they cannot borrow limitless amount of money at the same interest rate.

As opposed, the appeal from Physics relies on the kind of methodology frequently applied, mainly focused on an experimental basis. Thus, while economists often start with a model and after test what the data can say about such model, Physics tries to unfold the empirical laws which one later models. In other words, while the driving force in Physics is the quest for universal laws, economists are much more concerned about documenting differences. According<sup>25</sup> the need for an alternative approach stems from the fact that economic models cannot fully characterize the real behaviour of stock markets returns. This is especially true as it is widely recognized that the fundamental laws governing economic or financial systems were not yet completely understood. In addition to this, we may also stress that the empirical regularities exhibited by economic phenomena suggest that an important part of the social order may be incorporated in the Physics conceptual framework.

To sum up, we may say that the interest of physicists in economic/financial arena is due to four main factors<sup>26</sup>: (i) economic fluctuations affect everybody, which means that their implications are ubiquitous; (ii) everyone would be affected by a breakdown of the World-wide financial system; (iii) it is possible that methods and concepts developed in the study of fluctuation systems might yield new results; and, (iv) the existence of large data sets in economic/financial domain, which in some cases contains hundreds of millions of events.

### ***The Insight Through Econophysics***

During its short existence many empirical research has been conducted spanning different areas of knowledge in Economics and Finance. Space limitations however restrict us to a brief overview of both of them, centering the main focus on Finance Theory and in the interdisciplinary application of the concept of entropy. Nonetheless, a short summary of the state of the art is presented in order to provide a review of the kind of research performed. Specifically, topics in Economics include the distribution of income, the utility function, theories of how money emerges and the application of symmetry and scaling to the functioning of markets<sup>27-32</sup>. Among the important issues currently been debated in the World of Finance, the stock prices fluctuations have been recurrently addressed.

Several reasons have been advanced for that<sup>33</sup>. Firstly, it may be difficult to explain large fluctuations of asset prices based only on the information about the fundamental economic factors. This may lead to a lack of confidence on equity markets with the correspondingly consequences on their liquidity. Secondly, volatility is an important factor in determining the probability of bankruptcy of individual firms. Thirdly, price fluctuations can strongly influence the bid-ask spread. Therefore, the higher stock prices volatility, the higher that bid-ask spread will be; thus affecting market's liquidity. Fourthly, hedging techniques may be affected by the level of volatility, with the prices of insurance increasing with the volatility level. Fifthly, an increase in risk associated with a growing volatility may imply a reduced level of participation of investors in economic activity with adverse consequences on investment. Last, an increasing volatility may induce regulatory agencies to force firms to allocate a larger amount of capital to cash equivalent investments, which compromises the principle of capital efficient allocation. Additionally, the actual Worldwide crisis, which started in 2008, with the subprime credit default, has also drawn the attention to equity markets, their functioning, the degree of stock prices oscillations and their implications in real economy.

One major insight in Econophysics is the use of the Hurst exponents to determine whether asset prices might exhibit long-range correlations and thus may need to be described in terms of long-memory processes, such as the fractional Brownian motion (FBM). The idea of using the FBM to model asset price dynamics dates back to<sup>34</sup>. Since then, the Hurst exponent has been calculated for many financial time series, such as stock prices, stock indexes and currency exchange rates<sup>35,36</sup>. While studying market indexes an interesting feature appears to emerge<sup>36</sup>: large and more developed markets usually tend to be "efficient"

with a Hurst exponent close to 0.5, whereas less developed markets tend to exhibit long-range correlations. Accordingly, the multi-fractal dimension of stock markets returns has also been addressed<sup>37-40</sup>. In a recent work<sup>40</sup> concluded that the DAX data series shows very complex self-similar structures, which may escape to any unique multi-fractal description. One likely reason is that real financial data series are super-positions of series with different properties. In the same line<sup>41</sup>, found out that the multi-fractal structure of the traded volume of equities encompassing the Dow Jones 30 arises essentially from the non-Gaussian form of the probability density functions and from the existence of nonlinear dependencies.

Furthermore, the correlations among stock returns have also been addressed by means of the random matrix theory (developed in nuclear physics). It seems that the problem of interpreting the correlations among large amounts of spectroscopic data on the energy levels, whose exact nature is unknown, is similar of interpreting the correlations among different stocks returns. Therefore, with the minimal assumption of a random Hamiltonian, given by a real symmetric matrix with independent random elements, a series of predictions can be made. Some other examples concerning the application of this methodology to Finance problems can be found in<sup>42-44</sup>.

Another striking resemblance that unfolds when analyzing stock market volatility is its resemblance with the turbulence in fluids. Ref. [12, p. 88] addresses this as follows: "In turbulence, one ejects energy at a large scale by, *e.g.*, stirring a bucket of water, and then one observes the manner in which the energy is transferred to successively smaller scales. In financial systems 'information' can be injected into the system on a large scale and the reaction to this information is transferred to smaller scales – down to individual investors". Other studies dealing with turbulence include<sup>45-47</sup>. Moreover, the Omori law for seismic activity after major earthquakes has equally proved to be useful when understanding large crashes in stock markets<sup>48</sup>. Examples may ever continue with the application of some other concepts of Physics to financial markets, such as, the diffusion anomalous systems, whose general framework can be provided by the nonlinear Fokker-Planck equation<sup>17</sup>, etc. There is, indeed, a great deal of other empirical research using methods and analogies borrowed from Physics that space limitations prevent us to describe any further.

However, one that has drawn considerable attention for its apparent ability in describing stock market fluctuations is the concept of entropy. This notion was originally introduced in 1865 by Clausius to explain the tendency of temperature, pressure, density and chemical gradients to

disappear over time. Building on this, Clausius developed the Second Law of Thermodynamics which postulates that the entropy of an isolated system tends to increase continuously until it reaches its equilibrium state. Later, around 1900, within the framework of Statistical Physics established by Boltzmann and Gibbs<sup>49,50</sup>, it was defined as a statistical concept. Around the middle of the 20<sup>th</sup> Century, it found its way in engineering and mathematics, through the works of Shannon<sup>51</sup> in information theory and mathematics, and of Kolmogorov<sup>52</sup> in probability theory. Significant research has been done ever since with Shannon entropy providing the general framework for the treatment of equilibrium systems where short/space/ temporal interactions dominate.

However, many systems exist that do not satisfy the simplifying assumptions of ergodicity and independence. Some of these anomalies include<sup>53</sup>: (i) metaequilibrium states in large systems involving long range forces between particles; (ii) meta-equilibrium states in small systems (100-200 particles); (iii) glassy systems; (iv) some classes of dissipative systems, (v) mesoscopic systems with non-markovian memory. Due to the prevalence of these phenomena several entropy measures were derived. Among them, a most popular one was Tsallis entropy<sup>54</sup>, which constitutes itself as a generalized form of Shannon entropy. Although first introduced by Havrda and Charvát<sup>55</sup> in cybernetics and later improved by Daróczy<sup>56</sup>, it was Tsallis who has explored its properties and placed it in a physical setting. Hence, it is also known as Havrda-Charvát-Daróczy-Tsallis entropy.

Despite the debate generated over its meaning, for which the profusion of several mathematical constructions has certainly played a central role, entropy is commonly understood as a measure of disorder, uncertainty, ignorance, dispersion, disorganization, or even, lack of information. Recently<sup>57</sup>, has given it an econometric meaning, while considering that the entropy of an economic system is a measure of the ignorance of the researcher who knows only some moments' values representing the underlying population. Besides its multiples applications, entropy has recently started to be perceived as a consistent alternative to the standard-deviation, when assessing stock market volatility.

The underlying rationality is that, as a more generalized measure, entropy is able to capture uncertainty regardless of the kind of the empirical distribution evidenced by the data. This is especially so, as it is widely recognized that returns are usually non-normally distributed, where the application of the standard-deviation turns out to be unsatisfactory<sup>58</sup> emphasizes that as a function of many

moments of the probability distribution function, entropy considers much more information than the standard-deviation. Some of the main potentialities of this measure were summarized<sup>59,60</sup>: (i) it can be defined either for quantitative or qualitative observations; (ii) whereas entropy depends on the potential number of states of a distribution it is a result of the specific weight of each state, and; (iii) the information value is related to the respectively distribution function. For some empirical research concerning the application of entropy in the financial domain the interested reader is referred to<sup>15,18</sup>.

At the end, let us stress that this constitutes a brief presentation of the current efforts in Econophysics, mainly focused in the achievements in Finance Theory, which has only illustrative purposes and cannot be exhaustive.

### ***Opportunities Versus Limitations***

More than a decade passed since Econophysics formally emerged as a new discipline by the effort of H. E. Stanley and his colleagues, who are frequently referred as the Boston group. During this period an intense debate took place with a plethora of new papers and contributions arising from both domains – Physics and Economics/Finance – as the above-mentioned review illustrates. In order to assess the extent to whether this research has spread, its main implications, lessons learnt, major insights and whether the ultimate goals were achieved, an assessment is due. In doing so an examination of opportunities and limitations is conducted. The aim is to determine whether this approach is effective in describing complex economic/financial phenomena for which the traditional approach has proven to be insufficient.

By using the methodology applied in Physics which is extensively documented in literature, Econophysics can go forward and facilitate an integrated analysis based on the insights of both disciplines, with major benefits to the global comprehension of economic/financial phenomena. This may be done within the conceptual framework of Physics but without neglecting the concepts, theories and paradigms which have been demonstrated to be effective in the fields of Economics and Finance. With this kind of combination Econophysics becomes much more powerful and gives a full comprehension of the real implications of phenomena, which constitutes its main challenge/opportunity. This is strictly in line with the universality shown by the general use of Physics framework, which emphasizes that a certain order may exist in nature that makes this generality possible. Thus, there are reasons to believe that these approaches must be complementary and not opposed, as many times it appears to be. This leads to a common limitation frequently mentioned when addressing

this matter, which derives from the fact that if the traditional approach provided by Economics/Finance is not found fully satisfactory, thus everything else in the field must be disregarded. Although the mainstream work in Economics/Finance has repeatedly been criticized, some progress has been made to understand how Social and the World of Economics works, which must be considered when addressing these issues. The consequence of denying such achievements is a duplication of efforts that cannot be fruitful and at best can be a fragmented view of reality.

A pitfall that can arise when seeing through Econophysics, “glasses” is the tendency to recapitulate existing theories, already developed in Physics, without adding some fresh contributions. There may already appear some overlaps in previous works! The general idea is that Econophysics should adapt the conceptual framework of Physics instead of simply depicting it when processing economic/financial data. This is a corollary of the above-mentioned principle according to which Econophysics should combine both sciences. The extensive evidence of power-laws in Economics/Finance may illustrate that tendency. In fact, the ubiquitous nature of objects as fractal or self-similar has already been criticized<sup>61</sup>, who found out that the “scaling range of experimentally declared fractality is extremely limited”<sup>62</sup>, who alerted that “one should be careful in not seeing a power-law decline in each and every collection of data points with negative slope”.

Furthermore, the impossibility in performing experiments due to the kind of available data in Social Sciences constitutes an additional limitation that can be partly suppressed by applying the Physics methodology based on the quest for empirical laws. Even though experiments are still difficult to undertake, by focusing on what the observations can reveal about the phenomena and not trying to fit a particular model to the observed data, Econophysics can effectively contribute to the understanding of Economics and Finance World. A final shortcoming that should be pointed out is the tendency of econophysicists to develop theoretical models essentially based on the principles of Statistical Physics. If Physics can contribute to Economics and Finance a question that may arise is why not try other approaches based on different areas of Physics to see whether they can also give an effective contribution?

By highlighting these limitations we have no intention to particularly emphasize the potential flaws of Econophysics but rather to give a global perspective on its main drawbacks in order to raise a debate that can only be fruitful to the emergence of this new discipline. Accordingly, the discussion must be seen in a constructive way since it may generate future improvements in the field.

## Future Directions

Bearing on the above diagnostic some reflections arise. At the end, the question initially addressed of whether this would be a fashionable topic that turned out to be attractive for its innovative perspective but without any substance seems to be even more fundamental. In other words, what is in stake is if this is a new science or just a fashion trend that will gradually disappear over time when the subject is no longer a novelty. In view of the literature produced during these 15 years and on the advantage of the experimental method defended by econophysicists, much more focused on the data than on building a perfect elegant model, we might be tempted to answer that a new science is emerging. Indeed, we believe that the principles and the methodology, more than the particular techniques, might be applied (e.g., Hurst exponents; random matrix theory), that can really make the difference and provide new achievements. This idea is corroborated<sup>10</sup> through the neopositivist argument. In his view, Econophysics can be considered a separate discipline and not merely a branch of Economics since it proposes a different methodology based on a logical empiricism and on the idea that observations are the core of all scientific research. This is basically what the philosophical movement called “neopositivism” postulates: observational evidence is indispensable for the knowledge of the World.

Let us conclude by mentioning that even though Econophysics is an emerging discipline, it may aspire to be effective in describing Economics or Financial systems if its principles and methods can mature. Lot of work is going on in this field. The challenge is to see whether (or to what extent) this is achieved in the coming years. □

## References

1. J. Kisiel, S. Kowalski, E. Popiel, A. Ratuszna, B. Kozusznik and S. Mielimaka, *Physica A* **344** 340 (2004).
2. D. Grech, *Physica A* **344** 335 (2004).
3. H.E. Stanley, V. Afanasyev, L.A.N. Amaral, S.V. Buldyrev, A.L. Goldberger, S. Havlin, H. Leschorn, P. Maass, R.N. Mantegna, C.-K. Peng, P.A. Prince, M.A. Salinger, M.H.R. Stanley, G.M. Viswanathan, *Physica A* **224** 302 (1996).
4. E. Majorana, *Scientia* **36** 58 (1942).
5. D. Stauffer, *Journal of Modern Physics* **C 11** 1081 (2000).
6. E. Derman, *My Life as a Quant – Reflections on Physics and Finance*, (Wiley, Hoboken, New Jersey) (2004).
7. V. Pareto, *Cors d’Economie Politique*, (Lausanne, Paris), (1897).
8. L. Bachelier, *Ann. Sci. Ecole Norm. S. III* **17** 21 (1900).
9. A. Einstein, *Ann. Phys-Berlin* **17** 549 (1905).
10. C. Schinkus, *Physica A* (2010) doi:10-1016/j.physa.2010.05.016.
11. Z. Burda, J. Jukiewicz and M. Nowak, Is econophysics a solid science? Working Paper, Institute of Physics, (Jagellonian University, Krakow).
12. R.N. Mantegna and H.E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance*, (Cambridge University Press, Cambridge) (2000).
13. V. Plerou, P. Gopikrishnan, B. Rosenow, L.A.N. Amaral and H.E. Stanley, *Physica A* **279** 443 (2000).
14. S.M. Duarte Queirós, *Physica A* **344** 279 (2004).
15. S.M. Duarte Queirós, *Physica A* **344** 619 (2004).
16. L. Borland, *Europhysics News* **36** 228 (2005).
17. F. Michael, M.D. Johnson, *Physica A* 525 (2003).
18. S.R. Bentes, R. Menezes and D.A. Mendes, *Physica A* **387**, 3826 (2008).
19. S. Keen, *Physica A* **324** 108 (2003).
20. F.S. Lee and P. Downward, *J. Post Keynesian Econ* **23** 465 (2001).
21. E.F. Fama, *Journal of Finance* **25** 383 (1970).
22. P.J. Chung and D.J. Liu, *Quarterly Review of Economics and Finance* **34** 241 (1994).
23. Y-W. Cheung and L.K. Ng, *Journal of Empirical Finance* **5** 281 (1998).
24. N.A. Niarchos and C.A. Alexakis, *Applied Financial Economics* **8** 167 (1998).
25. G.L. Vasconcelos, *Brazilian Journal of Physics* **34** 1039 (2004).
26. H.E. Stanley, L.A.N. Amaral, D. Canning, P. Gopikrishnan, Y. Lee, and Y. Liu, *Physica A* **269** 156 (1999).
27. A. Dion’ysio, A. Heitor Reis and L. Coelho, *Physica A* 3862 (2008).
28. T. Souza, T. Domingos, *Physica A* **371** 492 (2006).
29. T. Souza, T. Domingos, *Ecological Economics* **58** 160 (2006).
30. L.A.N. Amaral, S.V. Buldyrev, S. Havlin, M.A. Salinger and H.E. Stanley, *Phys. Rev. Lett.* **80** 1385 (1998).
31. M.H.R. Stanley, L.A.N. Amaral, S.V. Buldyrev, S. Havlin, H. Leschhorn, P. Maass, M.A. Salinger and H.E. Stanley, *Nature* **379** 804 (1996).
32. H. Takayasu and K. Okuyama, *Fractals* **6** 67 (1998).
33. K. Daly, *Physica A* **387** 2377 (2008).
34. B.B. Mandelbrot and J.W. van Ness, *SIAM Review* **10** 422 (1968).
35. N. Vandewalle and M. Ausloos, *Physica A* **246** 454 (1997).
36. P. Grau-Charles, *Physica A* **287** 396 (2000).
37. B.B. Mandelbrot, *Journal of Business* **36** 394 (1963).
38. B.B. Mandelbrot, *Scientific American* **280** 70 (1999).
39. K. Matia, Y. Ashkenazy and H.E. Stanley, *Europhys. Lett.* **61** 4228 (2003).
40. A.Z. G’orski, S. Drozd and J. Speth, *Physica A* **316** 496 (2002).
41. L.G. Moyano, J. Souza, S.M. Duarte Queir’os, *Physica A* **371** 118 (2006).
42. L. Laloux, P. Cizeau, J.-P. Bouchaud and M. Potters, *RISK Mag.* **69** (1999).
43. S. Galluccio, J.-P. Bouchaud and M. Potters, *Physica A* **259** 449 (1998).
44. S. Galluccio and Y.C. Zhang, *Phys. Rev. E* **54** R4516 (1996).
45. S. Ghashghaie, W. Breymann, J. Peinke, P. Talkner, Y. Dodge, *Nature* **381** 767 (1996).
46. R.N. Mantegna and H.E. Stanley, *Nature* **383** 587 (1996).
47. R.N. Mantegna and H.E. Stanley, *Physica A* **239** 255 (1997).
48. D. Sornette, *Why Stock Markets Crash: Critical Events in Complex Financial Systems*, (Addison-Wesley, Redwood City), (1988).
49. L. Boltzmann, *Wien. Ber.* **76** 373 (1877).
50. J.W. Gibbs, *Elementary Principles in Statistical Physics*, (Yale University Press, New Haven) (1902).
51. C.A. Shannon, *Bell Systems Technical Journal* **27** 379; 623 (1948).
52. A.N. Kolmogorov, *Doklady Akademii Nauk* **119** 861 (1958).

53. C. Tsallis, F. Baldovin, R. Cerbino, P. Pierobon, Introduction to Nonextensive Statistical Mechanics and Thermodynamics, (2003) (cond-mat/0309093).
54. C. Tsallis, *Journal of Statistical Physics* **52** 479 (1988).
55. J. Havdra, F. Charv'at, *Kybernetika* **3** 30 (1967).
56. Z. Dar'oczy, *Information and Control* **16** 36 (1970).
57. A. Golan, *Journal of Econometrics* **107** 1 (2002).
58. E. Maasoumi and J. Racine, *Journal of Econometrics* **107** 291 (2002).
59. G. Philippatos and C. Wilson, *Applied Economics* **4** 209 (1972).
60. G. Philippatos and C. Wilson, *Applied Economics* **6** 77 (1974).
61. D. Avnir, O. Biham, D. Lidar and O. Malcai, *Science* **279** 39 (1998).
62. M. Gallegati, S. Keen and T. Lux e P. Ormerod, *Physica A* **370** 1 (2006).